DEFINITIONAL PROBLEMS WITH THE RATE OF RETURN CONCEPT

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Abstract

In most investments finance textbooks, the rate of return is defined in terms of the gain or loss based upon an initial investment (cash outflow) followed by a cash inflow at some later period. However, later chapters on short selling, options, and futures typically ignore this return definition. This paper speculates that this omission occurs since the conventional timeline return definition breaks down in these cases where there is no initial cash outflow, and because there is no well-known consensus on how to pedagogically resolve this issue . This paper, therefore, offers a surprisingly simple and even intuitive remedy using an internal rate of return (IRR) approach which provides a consistent definition for the rate of return using an "encumbrance" definition of investment throughout the major investment types.

INTRODUCTION

The concept of "return" is a basic, common theme in finance theory and finance textbooks. It is, in fact, one of the two legs upon which the most basic principle of the investment field stands, namely, that more risk entails more return. Most textbooks are careful to elucidate this concept by first explaining dollar returns and then by introducing the rate (%) of return. But why do they do this? The commonly-stated and valid reason, of course, is that at least *ex ante*, the use of a rate of return is to *compare* investments. In fact it is the foremost measure of an investment. Risk and liquidity are other ex ante measures used to gauge investments. *Ex post*, rate of return is the predominant measurement concern.

What is puzzling, however, is why the theme of risk and return gets so much less emphasis as the student goes through the chapters and why in particular the rate of return calculation gets so quickly abandoned in the later chapters on short sales, options, and futures trading. It would seem such a natural concept (and calculation) to retain in order to pedagogically unify the chapter presentations with the risk / return theme. Also, maintaining a more constant focus on the return calculation would provide useful and practical direction in the construction of investment performance guidelines involving more esoteric (derivative-based) investment portfolios.

As merely one example of many illustrating this "abandonment," Corrado and Jordan (2005, p.4) provide in their very first chapter the usual concept of the rate of return using a negative initial cash flow followed by a positive cash inflow later. However, on pp. 53 – 59 in their discussion of short sales, there is no rate of return

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The authors would like to thank Dr. Adrian Cowan and Mr. Brian K. Lee for helpful comments on earlier drafts of this paper.

calculation -- only *dollar* profits or losses are mentioned, although they *do* provide an end-of-chapter problem asking for a calculation of the rate of return. Similarly, in their chapter on options on p. 457 they calculate returns on *buying* calls and puts, but not on *writing* them. In their futures chapter, there is no mention of rates of return at all, only dollar gains or losses. Corrado and Jordan are hardly alone in this regard. A quick survey of other popular, mainstream investments texts reveals similar omissions of the rate of return concept in the short selling, options, and futures chapters.

For example, Bodie, Kane, and Marcus (2005) provide no explicit formula for a percentage rate of return through their first two chapters even though it is mentioned post hoc in a few examples. The holding period return (HPR) formula explicitly shows up for the first time in chapter 5 on p. 142. Earlier on p. 94 they provide a table with returns for a short-sale example without explaining their calculations, although by working backward one can determine the implicit equation being used. As for options, on p. 699 they show by example only the return on *buying* a put, but make no mention of the return on *selling* calls or puts, although they do calculate rates of return on a portfolio of stocks, bills, and options on p. 709. Otherwise, their discussion is couched mostly in terms of (dollar) profits. On p. 800 they do calculate a rate of return on futures using margin in the denominator, but the formula is not explicitly shown by itself – it occurs in the paragraph to illustrate the inherent leverage.

Reilly and Brown (2003, pp. 6 – 9) define HPR as the ending value of investment / beginning value of investment. On p. 126 they mention short sales but do not discus return percentages. On pp. 879-880 they *do* calculate percent returns on *buying* calls and puts, but not on writing them. Finally, they show no rate of return calculations for futures trades.

Hirt and Block (2003, p. 12) define the rate of return = (ending value - beginning value + income) / (beginning value), but their short sales discussion on p. 74 also omits any percentage return calculations. On p. 427, they use percentage return in a portfolio of covered calls but with no explicit formula. However, in a rare exception on p. 451, they use margin money in the rate of return formula denominator to find the percentage profit on *long* futures trades (and this is also done on p. 477 for long stock index futures), but they do not illustrate the *short* futures trade.

Alexander, Sharpe, and Bailey (2001, p. 3) show a return definition using wealth vs. value. On p. 31, however, in another notable exception, they do *calculate* short sale returns, although again the formula is not made explicit. There are no percentage return calculations on options, although on p. 642 there is an example for a *long* futures trade, but that example is tucked away in a paragraph and also has no corresponding example for a short futures rate of return. On p. 652 they show a table of futures returns, a rarity in investments texts.

Mayo (2006, p. 31) first mentions the calculation of return but then only in a margin paragraph context to compare the "percentage earned on cash purchase" = $(P_2 - \text{fee} + \text{income} - P_1) / P_1$ vs. the percentage earned on margin purchase $=(P_2 - \text{fee} + \text{income} - \text{interest}) / \text{equity}$. The HPR definition first formally appears on p. 136. There is no calculation of the percentage return earned on short sales on pp. 34-37, but he *does* discuss percentage returns on *buying* puts on p. 386 although not on *writing* put rate of return percentages. On p. 428 he calculates a percent return on *buying* futures based on margin in the return formula denominator, but he does not show the corresponding percent for *selling* futures.

Finally, Charles P. Jones (2004, p. 141) provides an initial definition of total return = yield + price change, and on p. 145 extends that with total return (TR) = (cash payments received + price changes over period) / (price at which asset purchased). He too provides no formulas or examples for percentage returns on short sales, options, or futures.

The problem causing this almost uniform omission of a percentage return on short sales, options (especially writing), and futures, it may be speculated, is that the nigh-well universal and conventional definition of rate of return involving an initial cash outflow followed by a later cash inflow does not appear to fit these investment situations. None of the investment finance texts nor general finance texts, undergraduate or graduate, have *formally or explicitly* shown how to resolve this predicament or how to justify the calculations they actually use. Yet it seems pedagogically incomplete for finance texts to omit these calculations of the rates of return for these other investment situations after initially explaining how important a role that the rate of return plays in the most fundamental, basic, and important rule of investments theory, namely, that higher rates of risk result in higher expected (percentage) *returns*.

The purpose of this paper, therefore, is to show how finance textbooks (and teachers) can apply a consistent calculation formula based on an expanded opportunity cost viewpoint of "encumbrances" that employs an internal rate of return calculation to resolve this issue. This new definition will be first applied to the traditional investment case to establish the general method, and then it will be applied to short sales, options, and futures contracts. A short conclusion will follow.

THE CONVENTIONAL RATE OF RETURN CALCULATION

Begin with the conventional case of buying a stock for, say, \$30 at time t=0. Let the stock price rise to \$33 at t=1, resulting in a well-defined, conventional, textbook holding period return of 10%. The formula, explicitly stated, is:

$$r = \text{net gain} / \text{initial investment}$$

where r is the "holding period return (HPR)", net gain includes any capital gains or other income distributions such dividends or coupons over the (arbitrary) investment period, and the initial investment is the cash outflow at time zero. The calculation yields

$$r = (\$33 - \$30) / \$30 = 10\%$$

This calculation can also be seen as the HPR necessary to produce a zero present value in an IRR sense as follows:

$$PV = 0 = + CF_0 + CF_1 / (1 + r) \tag{1}$$

wherein r is the HPR, the value of CFo is typically negative for buying investment assets, and CF_1 is typically positive. Solving for r gives:

$$r \text{ (of buying an asset)} = \mathbf{-} (CF_0 + CF_1) / \; CF_0 \tag{2}$$

so that in the example above for buying an asset

$$r = \mathbf{-} (- 30 + 33) / (- 30)$$
$$r = 10\%$$

Although this is the traditional formulation when an upfront cash outflow occurs, it is necessary to note that the sign on the return in formula (1) must be reversed in a "borrowing" situation or in any situation in which there exists a positive cash inflow at time zero such as short sales, writing options, and short futures. Therefore, equation (1) in such a borrowing situation would be re-written as:

$$PV = 0 = +CF_0 + CF_1 / (1 - r) \quad (3)$$

and so r would also be re-written as

$$r \text{ (of borrowing an asset)} = +(CF_0 + CF_1) / CF_0 \quad (4)$$

If margin is used, then the formula would calculate a return on equity (ROE) versus an return on (total) investment (ROI). For example, if the investor borrowed, say, \$15 and used only \$15 of his or her equity to buy the stock, then the ROI would still be \$3 / \$30 = 10% and the ROE would be (\$33 - \$15 - \$15) / \$15 = 20% (ignoring interest on the loan).

SOLUTION TO THE SHORT SALE RATE OF RETURN

In a short sale, the usual definition of the rate of return (ignoring any margin requirements or transactions fees for the moment), will confound students since there is no initial outflow of money and subsequent inflow of cash – in fact, just the opposite. The very few texts that actually do calculate a rate of return for this situation (such as the implicit example in BKM (2) or the more explicit example in ALEXANDER, SHARPE, AND BAILEY(1)), simply calculate (without defining) the rate of return as the ratio of the gain or loss on the short sale to the value of the stocks shorted (again, ignoring margin) so that:

short sale return = (profit or loss) / short sale

which in this case would be - \$3 / \$30 = - 10%. That is certainly a simple enough solution, but the problem is how to *justify* that calculation since the signs on the cash flows are reversed. In a more linguistic mode, how can one call the short sale an "investment" if nothing is being bought? Similar conceptual problems will occur with writing options and buying or selling futures, and so this paper proposes that all of these return calculations be put under a common rubric – that is, an internal rate of return format. So, for example, in the case of the short sale calculation of the rate of return, inserting the actual cash flow signs in the equation (3) yields

$$r \text{ (of a short sale)} = +(CF_0 + CF_1) / CF_0$$
$$= (+30 - 33) / 30 = -3 / 30 = -10\%$$

Note that the numerator was assigned a positive sign (outside the parentheses) because a short sale is really a borrowing situation. Although this calculation for the short sale rate of return is performed routinely in academic research and practitioner usage, the *pedagogical* problem addressed here is that the calculation is used *uncomprehendingly* in the face of the longstanding and definitional (time-line) implication that the return definition has a negative cash flow up front and assumes positive cash flows at the back end. This paper further proposes, however, to provide a *rationale* for using the short sale amount as the denominator in the return calculation (and for the subsequent amounts in the denominators of the options and futures returns. The intuition behind such a rationale is that what is needed is an expanded definition of the denominator as the initial amount of money "encumbered", not strictly the negative cash outflow upfront presumed in the conventional rate of return definition for "investments." Therefore, a new definition of return would be

$$HPR = (\text{net gain / encumbered funds}) \quad (3)$$

wherein encumbrance would then cover not just negative cash outflows or margin requirements but *any* other claim to assets such as the promises involved in trading options or futures. Any attempt at such justification, as far as is known, appears nowhere in finance textbooks. In other words, inputting the upfront positive $30 short-sale cash flow in the denominator of (2) versus the traditional definition of buying some investment like a bond or stock can be seen in another way; namely, that borrowing (shares) in a short sale transaction presumes the existence of borrowing capacity or the existence of funds sufficient to make the buyback at some point in time. Once the short sale is consummated, it reduces such borrowing capacity – that is, the short sale "encumbers" the investor.

If the investor puts up margin in a short sale (see, for example, (BKM, pp. 86-88) and (Alexander, Sharpe, And Bailey, p. 29)), then one would simply use the margin to calculate a return on equity (ROE) vs. a return on the total investment (ROI) – that is, the return would be modified to reflect the use of debt (another part of the more general encumbrance concept), but it would not change the general principle. [1] In the example above, the investor would short sell (borrow and sell) the stock at the current stock price of $30, but would then have to put up $15 (assuming a 50% margin requirement) in initial margin. Excluding earning any income on the margin then, if the price of the stock rose to $33, the investor's rate of return would be

$$r\,(ROE) = -(CFo + CF_1) / CFo$$

$$r\,(ROE) = -(-15 + 15 + 30 - 33) / (-15) = -20\%$$

where the negative $15 is the margin, the positive $15 is the recapture of margin, positive $30 is the short sale, and negative $33 is the short sale cover.

SOLUTION TO THE OPTION (WRITING) RATE OF RETURN

There is, of course, no problem in applying the conventional rate of return formula in defining option returns from ***buying*** calls or puts. For example, at time t=0 buy a call for $2 on stock ABC with exercise price of $120. Let the price of the stock rise to $126. Using equation (2), then the return from buying a call would be

r (of buying a call) = $-(CF_0 + CF_1) / CF_0$ which equals $-(-\$2 + \$126 - \$120) / (-\$2) = 200\%$, a well-known result which dramatically shows the leverage inherent in options. [2]

The problem occurs in ***writing*** a call (or put) wherein there is no traditional negative cash outflow ("investment") in the denominator of the returns definition. However, this is basically a borrowing situation, so the IRR borrowing methodology used above (equation 4) to obtain the short sale return can be extended to this case. Assume a call is *sold* with exercise price of, say, $120, but with a call premium of $2. Then let the stock price rise to $126 as before. The dollar return can be easily calculated as $120 - $126 + $2 for a $4 loss. In order to calculate the HPR, use equation (4) as follows:

$$r\,(\text{of a written call}) = +(CF_0 + CF_1) / CF_0$$

$$r = +(+\$2 - \$126 + \$120) / \$2 = -200\%$$

Thus, the call price or premium is the appropriate base of investment to use when calculating the rate of return on writing a call (as well as the appropriate base when buying the call). The *initial* investment basis or encumbrance is the call premium, even though one can *ultimately* lose more than that.

SOLUTION TO THE (LONG OR SHORT) FUTURES RATE OF RETURN

Finally, the case of futures trading involves no cash flow up front (at least for the moment assuming no margins), simply a promise to buy or sell at a later date, which is why it is technically incorrect to talk about "buying" or "selling" futures. Say the investor "goes long" at time t=0 or promises to buy 1 ounce of gold for, say, a futures price of $500 at time t=1. A return cannot be computed in the traditional sense because there is no cash outflow up front as specified in the conventional definition. However, this paper argues that the amount promised to buy or sell is the relevant encumbered base (an account payable, so to speak). So let the gold spot price rise to, say, $600 by time t=1. Applying the general IRR formula in this context using equation (2) gives

$$r \text{ (of going long)} = -(CF_0 + CF_1) / CF_0$$

$$r = -(-\$500 + \$600) / (-\$500)$$

$$r = +20\%$$

Therefore, the general principle in this context would be to define the denominator of a long futures HPR as the investment basis, or the promise to pay, or the futures price.

For the case of a short future, say the investor goes short or promises to sell a commodity for a futures price of $500 at time t=1. Then let the spot price be $600 at time t=1. As a result, the investor loses $100 going short. Again, conventionally, one cannot compute an HPR on the short future. However, if one defines a dollar return on the short future as losing $100, one can calculate a rate of return by using the promised sale (account receivable) as the investment base. Since this is inherently another borrowing situation, one would rely on equation (4) as follows:

$$r \text{ (of going short)} = +(CF_0 + CF_1) / CF_0$$

$$r = +(+\$500 - \$600) / \$500$$

$$r = -20\%$$

In summary, the short future return is calculated based on the investment basis or the $500 spot price that would be paid to make good on the promised sale.

Similar calculations could also be made using the margin base for returns on equity as in the case of short sales or options above.

CONCLUSION

The absence of a specific formula in investment applications without upfront investments has existed for decades in finance investments textbooks, and none of the texts has evidenced any awareness of the issue, much less proposed a resolution. Whatever the reasons for the failure to define and consistently use a return formula throughout all the investment types in the past, this paper puts forth a consistent and intuitive basis for now doing so. It is the author's hope that future finance textbooks will incorporate this broader definition of the rate of return based on an IRR calculation in order to provide students with a more consistent and reinforcing emphasis on the measurement of return.

ENDNOTES

[1]Corrado and Jordan (2005, p. 71) also calculate a rate of return for short sales in the same way on margin trades, using the margin as the investment basis. Unfortunately, they do not *explicitly* show how to do this.

[2]This is a well-known result and is usually covered in the textbooks except, as noted above, in Alexander, Sharpe, And Bailey (2001) and Jones (2004).

REFERENCES

Alexander, Gordon J., William F. Sharpe, and Jeffery V. Bailey, *Fundamentals of Investments*, 3rd edition, Prentice Hall, New Jersey, 2001.

Bodie, Zvi, Alex Kane, and Allen J. Marcus, *Investments*, 6th edition, McGraw-Hill, New York, 2005.

Corrado, Charles J. and Jordan, Bradford D., *Fundamentals of Investments: Valuation and Management*, 3rd edition, McGraw-Hill/Irwin, New York, 2005.

Hirt, Geoffrey A. and Stanley B. Block, *Fundamentals of Investment Management*, 7th edition, McGraw Hill / Irwin, New York, 2003.

Jones, Charles P., *Investments: Analysis and Management*, 9th edition, John Wiley & Sons, New Jersey, 2004.

Mayo, Herbert B., *Basic Investments: An Introduction*, 1st edition, Thomson / Southwestern, Ohio, 2006.

Reilly, Frank K. and Keith C. Brown, *Investment Analysis: Portfolio Management*, 7th edition, Thomson / Southwestern, Ohio, 2003.