The Profit Function: A Pedagogical Improvement For Teaching Operating Breakeven Analysis

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Abstract

This paper presents a graphical approach for showing elementary finance students how changes in operating leverage affect operating risk. It extends the traditional linear breakeven analysis model to develop the "profit function." First, profit functions for low and high operating leverage technologies are compared graphically under certainty. Then, a detailed exposition explains how to make graphical comparisons under uncertainty where operating risk is defined first to be the range of EBIT, and then where it's defined as the standard deviation of EBIT. The paper concludes by briefly discussing how the graphical approach can be applied also to illustrate differences in financial leverage, and the resulting differential effects upon risk.

INTRODUCTION

For decades, operating breakeven analysis, also called cost-volume-profit (CVP) analysis, has been used to analyze the relationship between sales volume and operating profitability. It determines the operating breakeven point, that level of unit sales where total revenue from sales just covers total (both fixed and variable) operating costs. Levels of unit sales above the operating breakeven point generate operating profits; levels below that point result in operating losses. Discussions of operating breakeven analysis appear in many introductory financial management textbooks, for example, Besley and Brigham (2000). A reasonably comprehensive recent review of CVP analysis is provided by Guidry, et al. (1998).

The purpose of this article is expositional; it presents a different way of visualizing breakeven analysis graphically. The next section reviews the standard approach to operating breakeven analysis. The following section introduces the profit function approach. Then the next section describes how profit functions can be used to compare operating earnings resulting from alternative technologies using either low or high operating leverage when there is no uncertainty in sales. With uncertainty in sales, the subsequent section uses the new approach to visualize the effect of changes in operating leverage on expected value and risk of EBIT, where risk is measured by the range of outcomes under uncertainty. The next section performs a similar analysis for a more restrictive case of normally distributed sales. The penultimate section briefly relates the approach of this study to the currently more conventional approach that focuses upon systematic risk rather than total risk. Finally, the last section describes briefly how the approach can be extended to illustrate financial leverage, and concludes with some additional discussion.

STANDARD OPERATING BREAKEVEN ANALYSIS

One strong advantage of the breakeven analysis approach is that the implications of changing sales volume on profit can be illustrated in a breakeven graph. In operating breakeven analysis, several different terms for "profit" can be used interchangeably, including "operating profit", "operating income", "EBIT", "earnings before interest and taxes", and "operating earnings". Linear breakeven analysis assumes that all costs can be categorized as fixed costs totaling F dollars per period or variable costs of v dollars per unit of output Q. Unit price is assumed constant at p dollars per unit. Then total revenue TR can be expressed as

$$TR = pQ \quad (1)$$

and total cost TC as

$$TC = F + vQ. \quad (2)$$

The contribution margin c is defined as the addition to profits, or operating income, from the sale of one additional unit of output. Thus c is the difference between price and variable cost:

$$c = p - v. \quad (3)$$

The breakeven output level Q_B is the output level at which total fixed operating costs are covered, and can be computed algebraically as

$$Q_B = F / (p-v) = F / c. \quad (4)$$

The standard linear breakeven graph (Figure 1, Panel A) shows the operating breakeven unit sales level Q_B as the intersection of two upward sloping lines on a space with units sold Q on the horizontal axis and both total revenue TR and total cost TC on the vertical axis. Many introductory texts, for example, Besley and Brigham (2000) and Keown, et al. (2002), include and discuss this graph. The total revenue line starts at the origin with a slope equal to the marginal revenue product, or unit price p. The total cost line starts from the level of total fixed costs F on the vertical axis with a slope equal to unit variable cost v. The vertical distance between the total revenue and total cost lines illustrates the level of operating profit or loss, with positive profit to the right of the breakeven level and negative profit, or loss, to the left.

Figure 1 here

Despite the assumptions of constant price and variable cost through the entire range of output, breakeven analysis is a useful first way for students to examine the effects of changing unit sales upon profit. We can remind students that within a limited range around the most likely sales level, it is appropriate to approximate the true nonlinear relationship with a linear one. When using comparative statics, the magnitude of resulting changes may not be exactly correct, but the direction of the change will be.

A related form of breakeven analysis, sometimes called "Sales Breakeven Analysis," uses dollar sales rather than unit sales. This approach may be more appropriate for firms with multiple products (which requires the assumption of constant product mix at different sales levels), or for outside analysts who do not have access to disaggregated unit sales information. All the following discussion can be applied to dollar sales breakeven analysis as well as unit sales breakeven analysis.

The Problem with Standard Breakeven Analysis

This standard breakeven graph is reasonably understandable for a single production process. However, if the firm wishes to compare several different technologies, each with a different total cost function, then the standard approach can become unnecessarily confusing for elementary students. Some texts such as Besley and Brigham (2000), actually compare two or three different technologies using two or three separate breakeven graphs. This paper describes an alternative graphical approach that facilitates visual comparisons on a single graph between technologies with differing total cost functions resulting from different levels of operating leverage.

THE PROFIT FUNCTION: AN ALTERNATIVE APPROACH

Profit Functions Defined

This alternative graphical approach specifies a profit function by subtracting total operating costs TC from total revenue T, where X denotes equivalently profit, EBIT, or net operating income,

$$X = TR - TC \qquad (4)$$

or the relationship between unit sales and net operating income, is obtained by combining (1) - (4):

$$X = -F + cQ. \qquad (5)$$

This profit function is illustrated graphically in Figure 1, Panel B, with operating profit X on the vertical axis and units sold Q on the horizontal axis. The profit function starts from the vertical axis at a negative amount equal to total fixed costs and then slopes upward with a slope equal to the unit contribution margin c, the difference between price and unit variable cost. The breakeven point Q_B is the point where the profit function crosses the horizontal axis, the level of zero profit. The profit function uses only one line to convey exactly the same information that traditional breakeven analysis graph requires two lines to convey.

Operating Leverage

Frequently we wish to compare the effects on operating profits of different production processes that use different values of fixed and unit variable operating costs. These different technologies exhibit different levels of operating leverage, with higher operating leverage indicated by higher levels of fixed costs accompanied by lower unit variable costs. Actually, there is not complete agreement upon how to define operating leverage. Some authors such as Besley and Brigham (2000) define operating leverage as the level of fixed costs associated with a production process. Others, such as Ross, Westerfield, and Jaffe (1999) actually define operating leverage as

((Change in EBIT) / EBIT) / ((Change in Sales) / Sales). (6a)

This measure is more commonly known as the Degree of Operating Leverage, or DOL. Using the term from microeconomics, the DOL is the elasticity of EBIT with respect to Unit Sales. For the linear profit function in (5) the DOL is computed as

$$DOL = Q_0c / (Q_0c-F). \quad (6b)$$

From (6b) it is clear that the DOL is a function both of the particular technology (its fixed and variable costs), and of a base output level Q_0. Since the DOL for a production process can vary as the unit sales benchmark varies, it is not a good measure solely of the operating leverage associated with a technology. Furthermore, the more commonly accepted definition of operating leverage increases as increases in fixed costs, does not adequately demonstrate the effect on EBIT of a change in sales. Scott (1998) recognizes this deficiency and advocates that operating leverage be measured by the ratio of unit variable cost to price. The discussion of the next section suggests that there is a tacit link between fixed and unit variable costs. An increase in operating leverage implies increased fixed costs accompanied by lower unit variable costs, and a higher unit contribution margin. In addition, in the classic derivation of a systematic risk approach focusing on operating and financial leverage, Rubinstein (1973) uses contribution margin to reflect operating leverage. Therefore, this study uses contribution margin as a measure of operating leverage.

ANALYSIS WITH NO UNCERTAINTY IN UNIT SALES

In the following discussion, subscripts L and H indicate respectively a low operating leverage technology and a high operating leverage technology. For a lower (higher) operating leverage profit function, operating profit X_L (X_H) is a function of fixed costs F_L (F_H), and unit variable cost v_L (v_H) which through c = p – v determines the contribution margin c_L (c_H). The two profit functions are,

$$X_L = -F_L + c_L Q, \quad \text{and } X_H = -F_H + c_H Q. \qquad (7)$$

which are displayed in Figure 2.

Figure 2 here

The slopes of the profit functions are the respective unit contribution margins. Students can readily see that higher levels of operating leverage result in more steeply sloped profit functions. The general reasoning is that higher operating leverage implies a higher level of fixed costs. However, no firm will voluntarily choose to increase fixed costs without an accompanying decrease in unit variable costs, which implies that increases in operating leverage imply an increase in the contribution margin, and a more steeply sloped profit function.

The respective breakeven levels of output are displayed in Figure 2 as Q_{BL} and Q_{BH}, where each profit function intersects the horizontal axis. At those points, operating profit is zero, as operating revenue is exactly offset by total operating costs.

The output level where the two profit functions intersect is called the indifference (or sometimes the crossover) point. At this point, labeled Q_I in Figure 2, operating income X_I is the same for both levels of operating leverage. The value of the indifference point is determined by setting the right hand sides of (7) equal to each other and solving for Q to get Q_I

$$Q_I = (F_H - F_L) / (c_H - c_L). \qquad (8)$$

If forecasted sales are to the right of this indifference point, then a more highly leveraged process will generate a higher level of operating earnings; conversely, if forecasted sales are to the left of the indifference point, the lower level of operating leverage will generate higher operating earnings. These comparisons under certainty will be extended conceptually to the expected (most likely) values of uncertain sales and operating earnings under different levels of operating leverage.

ANALYZING UNCERTAIN EBIT USING THE RANGE AS A MEASURE OF RISK

A even stronger pedagogical advantage of the profit function exposition over traditional TR/TC breakeven analysis occurs when operating leverage alternatives are examined under conditions of uncertainty. Giving elementary students a feel for analytical frameworks to handle uncertainty is an important contribution of the introductory finance course. There are different ways to demonstrate the effects of changes in operating leverage upon the risk of operating earnings.

The most easily understandable approach uses ranges of sales and operating earnings as proxies for the dispersions of each distribution of possible values. Figure 3 extends the analysis of Figure 2 to allow for uncertain values of sales, and the resulting uncertainty in operating earnings associated with different levels of operating leverage. In a number of texts, uncertainty in operating earnings is defined as business risk.

Figure 3 here

In Figure 3 on the horizontal axis, Q_{MIN}, Q_{EV}, and Q_{MAX} respectively indicate the lowest, most likely (or expected value), and highest values of sales attainable during a subsequent period. The uncertainty in sales is represented by the range, $Range_Q = Q_{MAX} – Q_{MIN}$. Now the profit functions can be used to transform a set of values of unit sales into values of operating earnings for cases of low and high operating leverage.

Starting with the lower operating leverage profit function X_L, we can locate the corresponding values of operating earnings, respectively, X_{MINL}, X_{EVL}, and X_{MAXL}. The range of low operating leverage operating earnings is $Range_{XL} = X_{MAXL} - X_{MINL}$. Therefore, we can say that the given risk (uncertainty) in sales, RangeQ, combined with a level of operating leverage represented by the profit function XL, determine $Range_{XL}$, the business risk associated with the low operating leverage production process.

In parallel fashion, we can trace out the minimum, most likely, and maximum values of operating earnings X_{MINH}, X_{EVH}, and X_{MAXH} for the high operating leverage profit function, and the range is just $Range_{XH} = X_{MAXH} - X_{MINH}$. These ranges are displayed in Figure 3; it's easy to see from the graph that the technology with higher operating leverage results in a higher range of possible values of EBIT. With very little mathematics, students can see that the dispersion of possible values of operating earnings, for a given dispersion in unit sales, increases as operating leverage increases from profit function X_L to profit function X_H. In other words, increases in operating leverage cause increases in business risk.

The general chain of reasoning is as follows: operating leverage , fixed costs , unit variable costs , contribution margin , profit function slope , $Range_X$, and thus business risk . The only link in the reasoning chain that requires an external assumption is that fixed cost increases imply reductions in unit variable cost. Recall from earlier discussion, no firm would ever voluntarily increase fixed costs without a resulting increase in unit variable cost; if unit variable cost did not decrease, profit would be lower than before at all levels of output.

So, an increase in operating leverage always results in an increase in business risk, as measured by the variability of operating earnings. What about the most likely or expected value of operating earnings? Consistent with the certainty case above, if the most likely value of sales Q_{EV} is to the right of the indifference point Q_I, then increases in operating leverage will increase the most likely value of operating income. If the most likely value of sales is to the left of the indifference point, increases in operating leverage actually reduce the most likely value of operating earnings. No firm would realistically consider increasing business risk without an accompanying increase in expected return, here measured by the most likely value of operating earnings.

ANALYZING UNCERTAIN EBIT USING NORMALLY DISTRIBUTED SALES

This section discusses a slightly more advanced framework for the relationship between operating leverage and business risk that may not be appropriate for some introductory students. It specifies that the distribution of sales can be approximated by a normal distribution, using the approach originated by Jaedicke and Robichek (1964). This approach uses the convenient property of normal distributions that all points on a distribution can be summarized by only two parameters, the mean and the standard deviation.

For a distribution of uncertain future values of unit sales Q with expected value E(Q) and standard deviation $ó_Q$, a profit function with fixed costs F and contribution margin c will generate a distribution of operation earnings with a mean of

$$E(X) = -F + c\,E(Q) \qquad (9)$$

and a standard deviation of

$$\sigma_X = c\,\sigma_Q. \quad (10)$$

In this framework, business risk, the dispersion in operating earnings, is measured by the standard deviation of operating earnings, instead of the range.

When uncertainty exists, instead of a single value for sales, we specify a probability distribution of possible future sales values. We can then readily observe how the distribution of possible sales values is transformed through the profit function into a distribution of possible values of operating earnings. As the slope of a profit function increases, reflecting a higher amount of operating leverage, the resulting operating earnings distribution becomes more dispersed, reflecting the higher variability of operating earnings accompanying the increase in operating leverage. This graphical approach also reinforces the insight of Scott (1998), who observed that increases in operating leverage are more accurately reflected by increases in the unit contribution margin (which increases the slope of the profit function), rather than the traditionally stated increases in fixed operating costs.

Figure 4 here

For a given distribution in unit sales, Figure 4 illustrates two distributions of operating earnings corresponding to two different levels of operating leverage. The low operating leverage technology generates a distribution of EBIT with smaller total risk σ_{XL} and lower expected value $E(X_L)$, and the higher operating leverage technology results in an EBIT distribution with higher total risk σ_{XH} and expected value $E(X_H)$. As discussed earlier, increases in operating leverage, reflected by an increase in the unit contribution margin c, always increase the risk of EBIT. However, increases in the expected value of operating earnings will only occur if the forecasted expected value of sales is to the right of the indifference point.

OPERATING LEVERAGE AND SYSTEMATIC RISK

The previous two approaches to relating operating leverage to risk define risk as a total risk concept. Brigham and Daves (2002) refer to this type of risk as "stand-alone" risk, since any diversification benefits are ignored. Because of the predominance of the capital asset pricing model over the past few decades, it is more common to discuss risk in terms of systematic (beta) risk instead of total (sigma) risk.

However, at the elementary level, the focus of the graphical illustrations in this paper has been involved with how changes in leverage result in changes in risk, with less emphasis upon the actual values of risk. However, it is well known that for any asset A, its beta β_A and sigma σ_A risk are related as follows,

$$\beta_A = r_{AM}\,\sigma_A / \sigma_M, \qquad (11)$$

where r_{AM} and σ_M denote respectively the correlation of asset A with the market and the standard deviation of a market index. Therefore, other things equal, any increase in total risk will be accompanied by a corresponding increase in systematic risk. Therefore, conclusions regarding the effects of operating leverage upon risk using the elementary graphical approach of this paper are entirely consistent with those implied by such studies as Rubinstein (1973), and those reviewed in Li and Henderson (1991) that examine systematic risk instead of total risk.

EXTENDING THE APPROACH FOR FINANCIAL LEVERAGE

So far, the total risk graphical approach to describing the effects of leverage changes upon risk and return has been limited to operating leverage. It's very easy to extend this graphical approach to include financial leverage.

Figure 5 here

Figure 5 displays how operating and financial leverage together combine to transform a sales distribution into an earnings per share distribution. Earnings per share EPS are defined as

EPS = (Net Income) / (Common Equity)

Just as the operating leverage can be considered to transform a distribution of unit sales Q values into a distribution of values of operating earnings X, financial leverage can be considered to transform a distribution of values of operating earnings X into a distribution of values of earnings per share EPS. Extending Figure 4, which showed the transformation of a unit sales distribution into EBIT distributions, Figure 5 illustrates both transformations sequentially; sales into operating earnings and operating earnings into earnings per share. Some texts such as Van Horne (2000) define and discuss this latter transformation of EBIT into EPS as "EBIT/EPS analysis." Alternatively, some texts such as Brigham Gapenski and Daves (1999) describe a linear transformation of EBIT into Return on equity ROE, by dividing net income by common equity. Whether shareholder welfare is measure by EPS or by ROE, increases in financial leverage always result in increases in risk, the dispersion of possible values of both EPS and ROE.

CONCLUSION

This paper has outlined an exposition of operating leverage that may be easier for less mathematically inclined students to see graphically the effects of changes in operating leverage upon business risk. The approach relies upon specifying risk as a total risk measure rather than a systematic risk measure. Using the range of EBIT as a measure of dispersion, students can see exactly why operating leverage increases always result in increases in business risk. In addition, they can see exactly under what conditions increases in operating leverage result in increases in the expected value of EBIT; sometimes increasing operating leverage can decrease the expected value of EBIT. Although the major focus of the paper is on operating leverage, the paper adds a few comments regarding extension of the approach to analyze the effects on EPS or ROE of changing financial leverage.

REFERENCES

Besley, Scott, and Eugene F. Brigham (2000). *Essentials of Managerial Finance*, 12th Edition, Orlando, FL, Dryden Press.

Brigham, Eugene F. and Phillip R. Daves (2002). *Intermediate Financial Management*, 7th Edition, Orlando, FL, Dryden Press.

Brigham, Eugene F., Louis C. Gapenski, and Phillip R. Daves (1999). *Intermediate Financial Management*, 6th Edition, Orlando, FL, Dryden Press.

Guidry, Flora, James O. Horrigan, and Cathy Craycraft (1998). "CVP Analysis: A New Look, *Journal of Managerial Issues*, Spring, pp. 74-85.

Jaedicke, R. K., and A.A. Robichek (1964). "Cost-Volume-Profit Analysis under Conditions of Uncertainty," *Accounting Review*, October, pp. 917-926.

Li, Rong-Jen, and Glenn V. Henderson, Jr. (1991). "Combined Leverage and Stock Risk," *Quarterly Journal of Business and Economics,* Winter 1991; 30,1, pp. 18-39.

Ross, Stephen A., Randolph W. Westerfield, and Jeffrey Jaffe (1999). *Corporate Finance*, 5th Edition, Boston, MA, McGraw-Hill.

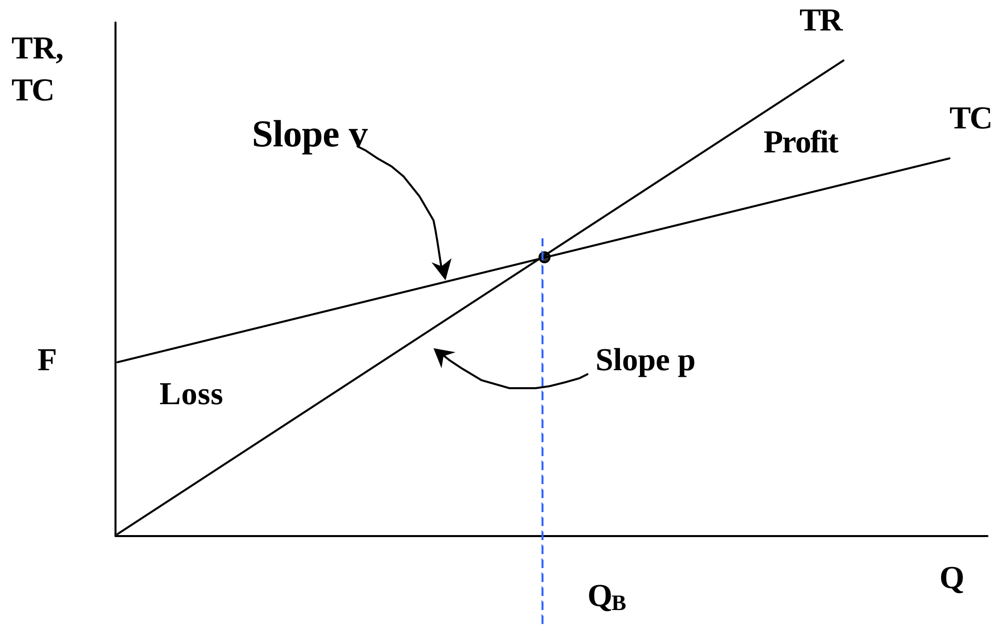
Rubinstein, Mark E. (1973). "A Mean-Variance Synthesis of Corporate Financial Theory," *Journal of Finance*, March, 28, 1, pp. 167-181.

Keown, Arthur J., John D. Martin, J.William Petty, and David F. Scott (2002). *Financial Management: Principles and Practices*, 9th Edition, Upper Saddle River, NJ, Prentice-Hall.

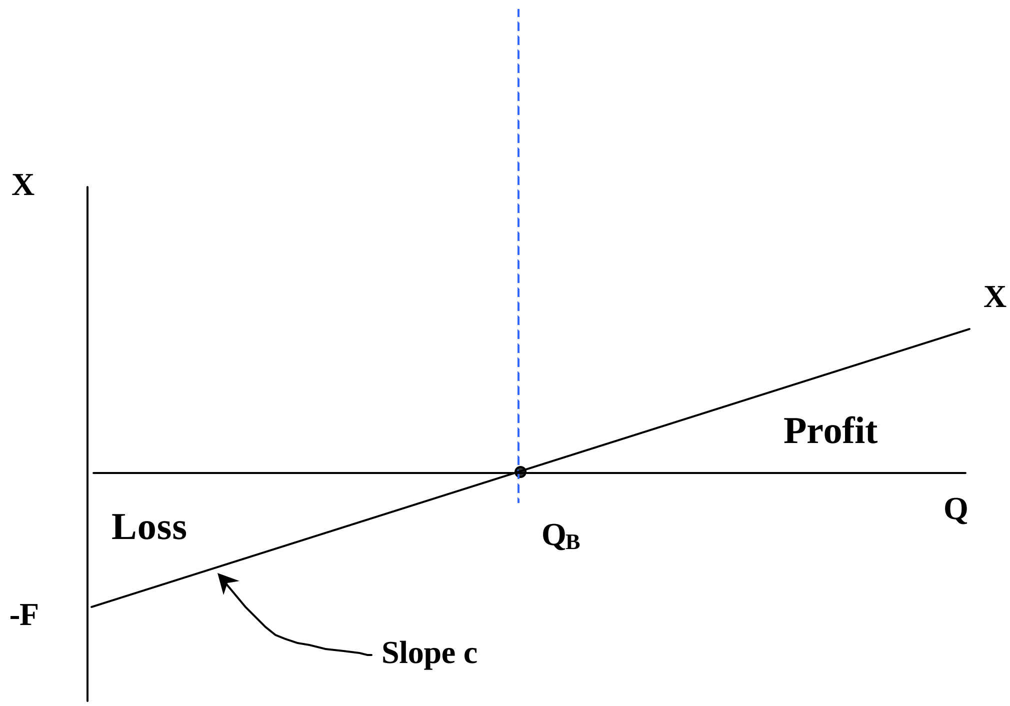
Scott, Carole E. (1998). "Insights into Operating and Financial Leverage," *B>Quest (Business Quest)*, http://www.westga.edu/~bquest/1998/, accessed September 3, 2000.

Van Horne, James C. (2002). *Financial Management and Policy*, 12th Edition, Upper Saddle River, NJ, Prentice-Hall.

FIGURE 1. TRADITIONAL BREAKEVEN ANALYSIS VS PROFIT FUNCTION ANALYSIS.

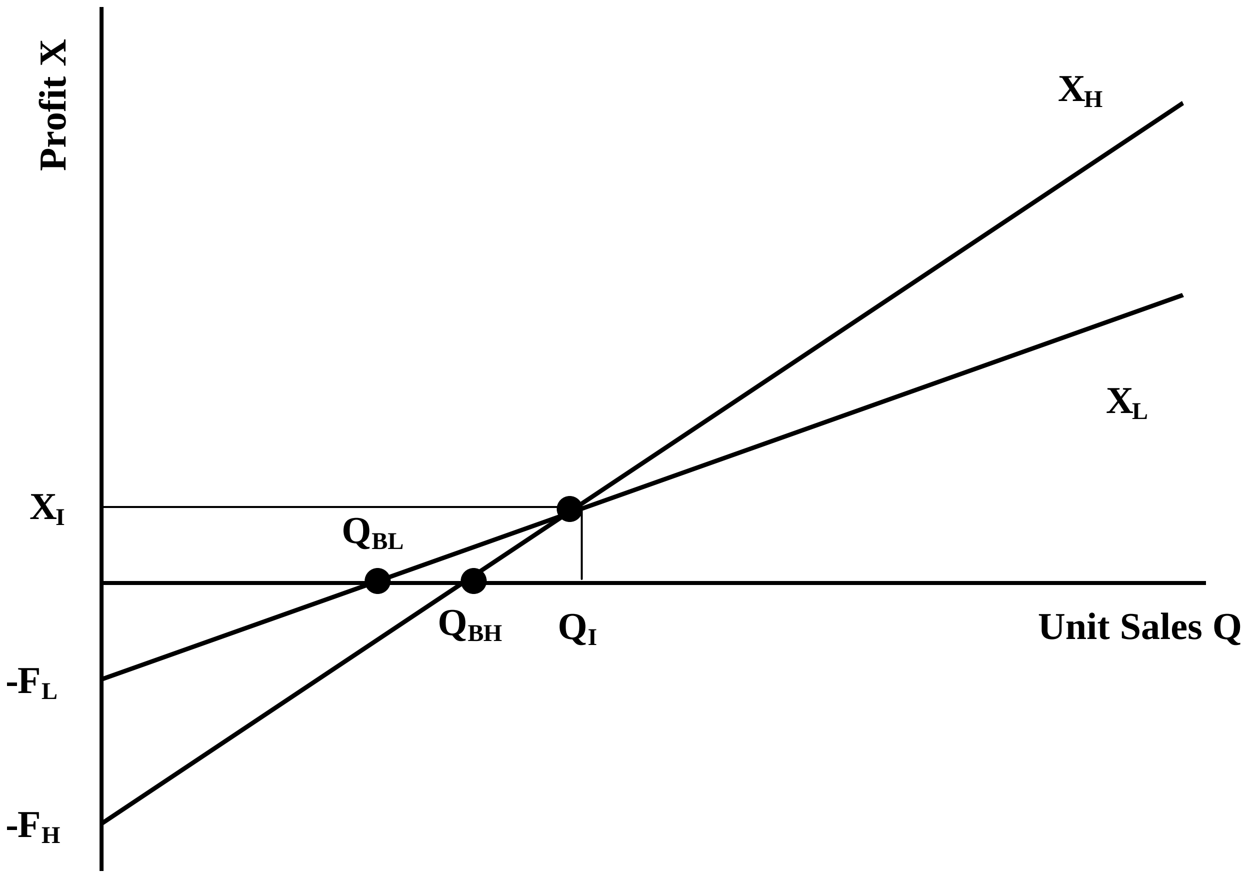


Panel A. Traditional Operating Breakeven Analysis Diagram. TR is Total Revenue, TC is Total Cost, Q is Unit Sales, F is Fixed Costs, v is Unit Variable Cost, p is price, and Q_B is the Breakeven Unit Sales value.



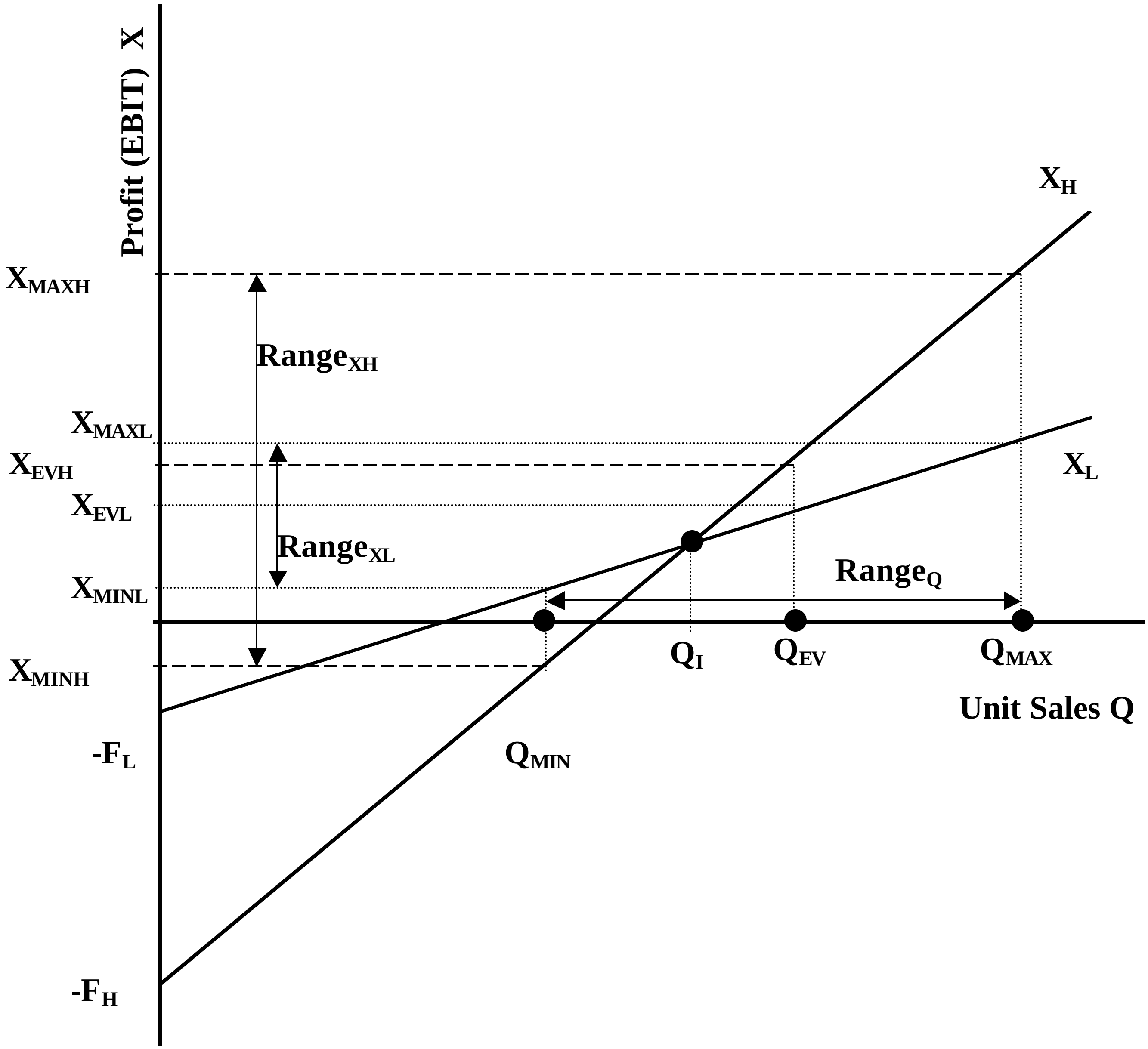
Panel B. Profit Function Analysis Diagram. X is profit, or operating earnings, or EBIT. c is unit contribution margin, the slope of the profit function. Q, Q_B, and F are defined in Panel A.

FIGURE 2. PROFIT FUNCTIONS: LOW AND HIGH OPERATING LEVERAGE



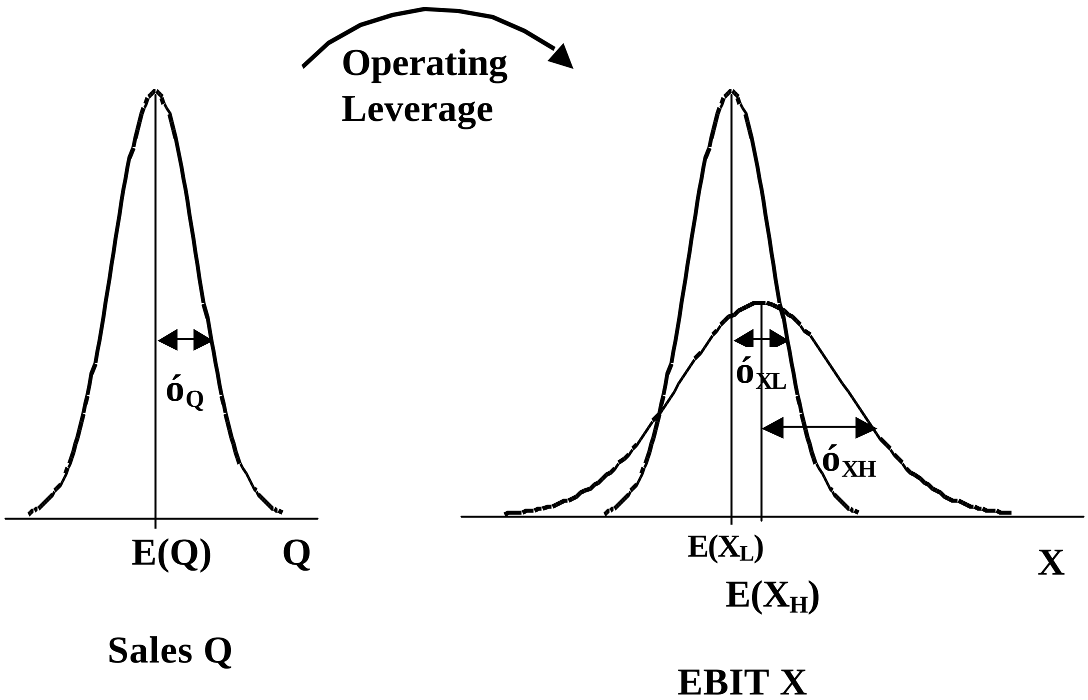
Profit Functions X_L and X_H for Low and High Operating Leverage with No Uncertainty in Sales. Breakeven points Q_{BL} and Q_{BH}, and Indifference Level Output Q_I, where both X_L and X_H have the same Operating Income X_I.

FIGURE 3. PROFIT FUNCTION ANALYSIS OF OPERATING LEVERAGE WITH UNCERTAIN SALES: RISK MEASURED AS RANGE



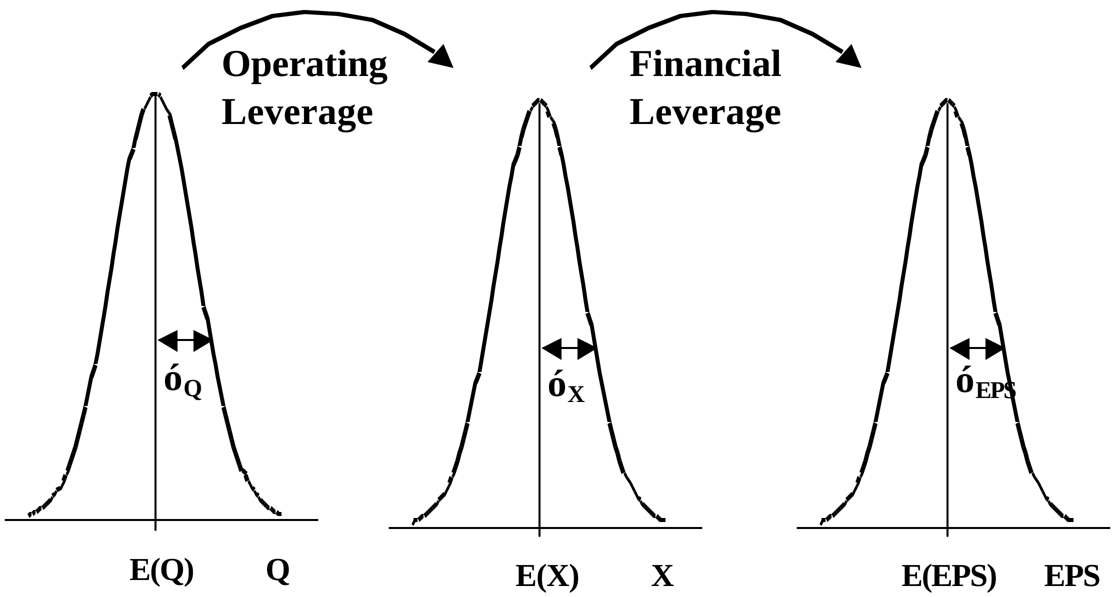
For a given range of unit sales $Range_Q$, higher oplev profit function X_H results in a larger EBIT range $Range_{XH}$ than $Range_{XL}$ which uses the lower oplev profit function X_L. Increasing oplev always increases risk. And, in this case, since the given expected unit sales Q_{EV} is greater than the indifference level Q_I, higher oplev expected EBIT X_{EVH} is greater than the lower oplev expected EBIT X_{EVL}. Increasing oplev increases (decreases) expected return if expected sales is greater (less) than the indifference level of sales.

FIGURE 4. LOW AND HIGH OPERATING LEVERAGE; RISK MEASURED AS STANDARD DEVIATION



For a given sales distribution Q with expected value E(Q) and standard deviation σ_Q, a low operating leverage technology generates an EBIT distribution X_L with an expected value $E(X_L) = - F_L + c_L E(Q)$ and standard deviation $\sigma_{XL} = c_L \sigma_Q$ that are respectively lower than $E(X_H) = - F_H + c_h E(Q)$ and $\sigma_{XH} = c_h \sigma_Q$ of the distribution X_H generated by an higher operating leverage technology. In this illustration, standard deviation is the risk measure; similar results occur if range is the risk measure, as it was in Figure 3.

FIGURE 5. OPERATING AND FINANCIAL LEVERAGE COMBINED



The distribution of earnings per share EPS is determined by three factors, the distribution of unit sales, operating leverage, and financial leverage. In the first step, the sales Q distribution and operating leverage determine the operating earnings X distribution. In the following step, the operating earnings distribution and financial leverage determine the distribution of EPS. Instead of an EPS distribution, a ROE distribution could be used to illustrate the same effects of financial leverage upon risk and return facing the shareholders.